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Subtraction method for NLO corrections in Monte-Carlo event generators for Z boson production

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ABSTRACT: We use a subtraction method to construct NLO corrections in a Monte-Carlo event generator for the case of vector boson production in Drell-Yan processes. Our calculations are carried out both for the Bengtsson-Sjöstrand-van Zijl (BSZ) algorithm and for a modified algorithm proposed by Collins. In the case of the modified algorithm, we compute the relation between the parton distribution functions and the ones in the $\overline{\mathrm{MS}}$ scheme; this relation is the same as the corresponding relation for DIS. For the BSZ algorithm, we show that there is no simple relation.

KEYWORDS: QCD, NLO Computations, Hadronic Colliders.

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Contents

1. Introduction

In an earlier paper by Collins [1], a subtraction method was introduced to consistently take into account next-to-leading order (NLO) terms for deep-inelastic scattering in a Monte-Carlo event generator. In that paper, the method was applied to the photon-gluon-fusion process in an event generator that uses the algorithm constructed by Bengtsson and Sjöstrand (BS) [2] for initial-state showering; such event generators are PYTHIA [3], LEPTO [4] or RAPGAP [5].[1] In this paper, we apply this method to massive vector boson production (W or Z production) in hadron-hadron collisions. The method contrasts with previous methods, e.g., [6], at incorporating NLO corrections by a reweighting of the events generated by showering from the LO matrix elements. The subtraction method is intended to be applicable to all the perturbative parts of an event generator, to calculate non-leading corrections to the hard scattering and the showering.

As described in [1], an event generator using this subtraction method to correct the hard scattering generates two classes of events. One class is obtained from the

[1] The same methods can be applied to event generators like HERWIG that use a different algorithm, but the details of the calculation will be different.

LO parton-model process by showering the initial and final state quarks, exactly as before. The second class of events is generated by starting with an NLO subprocess and showering the partons, again exactly as before, but with one exception, the exception being that the hard cross section for the subprocess is equipped with a subtraction that correctly compensates the double counting between the two classes of events. This removes the part of the NLO term that is included in the combination of the LO parton model and showering, and it cancels the singularity in the NLO contribution to the cross section.

We will use two different algorithms for the parton showers. One algorithm is due to Bengtsson, Sjöstrand and van Zijl [2] and is used in PYTHIA and RAPGAP. As explained in ref. [1], the parton density functions (pdf's) to be used in the event generator are not those of a standard scheme, but are specific to the showering algorithm. We will actually find that there is no simple relation whatever between the parton densities to be used with this algorithm and the $\overline{\rm MS}$ pdf's, and that they are therfore also different from the pdf's needed for the corresponding algorithm [7] in DIS. This happens because of the changes made by the showering algorithm to the parton model kinematics: the changes are different from those in DIS, and correlate the kinematics of the partons from each of the incident hadrons. The second algorithm is the one defined in [1] with the specific aim that the relevant parton kinematic variables do not get changed by the showering. For this algorithm, we will show that the pdf's for the hadron-hadron induced process are the same as in DIS, although both differ from the $\overline{\rm MS}$ definition.

The NLO corrections for vector boson production come from two different subprocesses: gq and $q\overline{q}$. In this paper we derive the results for gq subprocesses; this is an important correction which is not necessarily suppressed by the factor of α_s if the gluon density is larger than the quark densities. The generalization to $q\overline{q}$ subprocess will encounter some complications because of the need to treat soft gluon effects. So we defer this to the future.

In section 2, we describe the treatment of vector boson production in an event generator. Then we compute the effect of combining the LO cross section with the order α_s part of the shower; this will be needed as the subtraction term in the NLO calculation. We present the calculation for Z bosons, and later, in section 4, specify the changes needed to treat the production of W bosons.

In section 3, we carry out the subtraction from the NLO matrix elements to get the resulting NLO hard differential cross-sections. The pdf's in these formulae are not in $\overline{\rm MS}$ scheme, so we show how to relate them to the ordinary $\overline{\rm MS}$ pdf's. For the new algorithm, we find that the relationship between the pdf's is the same as was found with deep-inelastic scattering in [1]; while for the BSZ algorithm we show that no simple relation appears to be possible.

Finally, we summarize and discuss our results in section 5.

2. Monte-Carlo algorithms

2.1 QCD improved parton model and vector boson production

Our calculations are based on the QCD improved parton model, which shows that a hard scattering process initiated by two hadrons is the result of an interaction between one parton (quark or gluon) from each of the incoming hadrons [8].

We consider Z boson production in a collision of two hadrons A and B at a center-of-mass energy $\sqrt{s}$. The leading-order subprocess is $q\overline{q} \to Z$. As in figure 1, the resulting cross section $d\sigma_{AB}^{\mathrm{LO}}/dy_0$ for producing a Z boson of rapidity y_0 is obtained by weighting the subprocess cross section with the parton distribution functions $f_i(x_a)$ and $f_{\overline{i}}(x_b)$, and summing over all quark-antiquark combinations in the beam and target:

$$\frac{d\sigma_{AB}^{\mathrm{LO}}}{dy_0} = K \sum_i (A_i^2 + V_i^2) f_{i/A}(x_a, M_Z^2) f_{\overline{i}/B}(x_b, M_Z^2)\,. \tag{2.1}$$

The parton momentum fractions are written as

$$x_a = \frac{M_Z}{\sqrt{s}} e^{y_0}\,, \qquad x_b = \frac{M_Z}{\sqrt{s}} e^{-y_0}\,. \tag{2.2}$$

The parton-level cross-section results in the factor $K(A_i^2 + V_i^2)$, where

$$K = \frac{\sqrt{2}\pi G_F \tau}{3}\,, \qquad \tau = \frac{M_Z^2}{s} = x_a x_b\,, \tag{2.3}$$

and

$$A_i = T_i^3\,, \qquad V_i = T_i^3 - 2Q_i \sin^2\theta_w \tag{2.4}$$

come from the axial and vector couplings in the electro-weak interaction [8]. Finally, $f_{a/A}(\xi, \mu^2)$ is the number density of quarks of flavor a in hadron A at fractional momentum ξ and a renormalization/factorization scale μ.

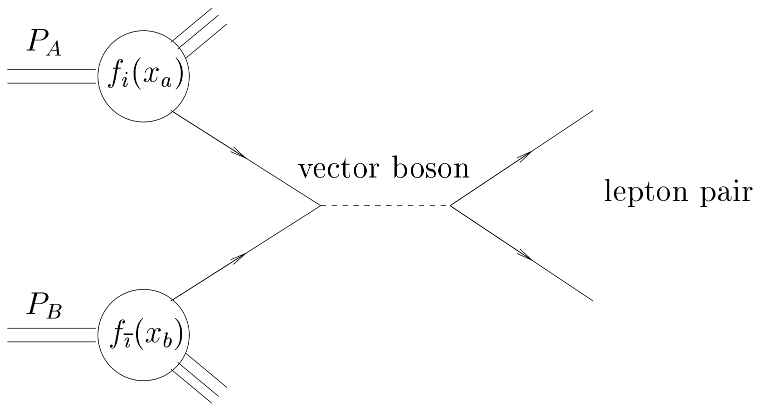

Figure 1: Vector boson production in the Drell-Yan model.

In the usual "matrix element" approach, the variable y_0 is exactly the rapidity of the Z boson. This follows from the approximation of giving the incoming partons zero transverse momentum and virtuality. However, in a Monte-Carlo event generator, the quark and antiquark are given their correct kinematics. In that case, the variable y_0 is not exactly the rapidity of the Z boson; its precise definition is $\frac{1}{2}\ln(x_a/x_b)$, where x_a and x_b are the fractional longitudinal momenta of the incoming partons. This is a variable that is well-defined in the generation of a particular event, but that is not necessarily measurable from the final state of the event. In the context of an event generator, eq. (2.1) must therefore be reinterpreted, not as the lowest-order approximation to the physical cross section for a particular Z-boson rapidity, but as the cross section for events that inside the program have a certain value for the variable y_0. Of course, the motivation for using the variable y_0 is that it approaches the true rapidity y in the limit that the parton transverse momenta and virtualities approach zero.

Since the Z's decay width is small compared to its mass M_Z, it is sufficient to compute the production cross sections of the bosons. The actually measured cross sections of leptons are computed by multiplying the cross sections by the appropriate branching ratios.

With full perturbative QCD corrections taken into account, eq. (2.1) gets replaced by the factorization formula

$$\frac{d\sigma_{AB}}{dy} = \sum_{i,j} \int d\xi_i \, d\xi_j \, f_{i/A}(\xi_i, \mu^2) f_{j/B}(\xi_j, \mu^2) \frac{d\hat{\sigma}_{ij}}{dy}, \tag{2.5}$$

where y is the rapidity of the Z boson, the ξ's are the momentum fractions of the incoming partons, and $d\hat{\sigma}_{ij}/dy$ is a suitably constructed hard scattering cross section. Now the sum is over all pairs of parton flavors (quarks and gluons). The formal domain of validity of eq. (2.5) is for the inclusive cross section in the asymptotic 'scaling' limit, analogous to the Bjorken limit in DIS, $s \to \infty$ with τ and y fixed. As is well-known, one effect of higher-order perturbative corrections is the production of vector bosons at large transverse momentum (q_T). The discussion above, about the distinction between the variables y and y_0, alerts us that care will be needed in our application of the factorization formula in an event generator. In effect, the quantitative interpretation of eq. (2.5) gets modified in current MC event generators; notably the lowest-order formula eq. (2.1) is no longer for $d\sigma/dy$ but for $d\sigma/dy_0$. Effectively, when only LO hard scattering is concerned and when q_T is small, the foundation of an event generator's algorithm is an appropriate modification of eq. (2.5) that embodies the same physics. However, when large q_T is concerned, we have to be careful about the difference between y and y_0 for all orders of hard scattering, including LO.

2.2 Parton-shower algorithm

Now we describe the initial-state shower algorithm for vector boson production used in PYTHIA, LEPTO or RAPGAP, as described in [2].

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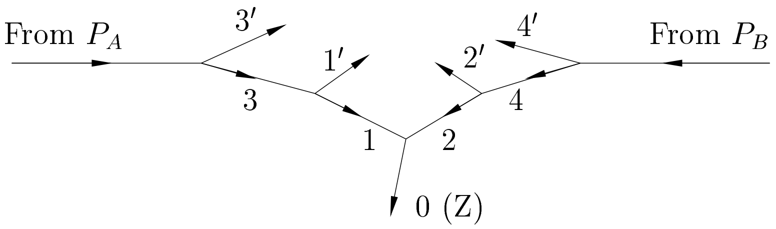

Figure 2: Initial-state parton shower for vector boson production.

In figure 2 is symbolized an example of initial-state parton showering for Z production. The showering algorithm generates partons with certain flavors and virtualities; it also generates the splitting variables z_{2n+1}, z_{2n} for each branching, and an azimuthal angle for the transverse momentum of each branching. The momentum fraction of each space-like line is computed as

$$\xi_{2n+1} = \frac{\xi_1}{\prod_{i=0}^{n-1} z_{2i+1}}\,, \qquad \xi_{2n} = \frac{\xi_2}{\prod_{i=1}^{n-1} z_{2i}}\,, \tag{2.6}$$

so that $\xi_3 = \xi_1/z_1$, $\xi_4 = \xi_2/z_2$, etc. Since the z's are generated numerically, by an algorithm explained below, eq. (2.6) gives the values of the momentum fraction variables for all the lines, except for ξ_1 and ξ_2. These values are defined to be x_a and x_b, which are generated according to the probability distribution corresponding to the lowest order cross section, eq. (2.1).

As in [2], the "$\hat{s}$ approach" is used to relate the splitting variables z_i to the parton 4-momenta. This is done by requiring that

$$\hat{s}_{ij} = \xi_i\, \xi_j\, s \tag{2.7}$$

both at the hard scattering and at any lower scale in backward showering, where ξ_i and ξ_j are of the two resolved partons. This means that the total $\hat{s}_{ij}$ has to be increased by a factor of $1/z$ in the backward evolution; this defines the relation between z and parton kinematics. For instance, in figure 2, if line 1 has the highest virtuality, then $z_1 = (p_1 + p_2)^2/(p_3 + p_2)^2$.

The part of the algorithm used in an event generator that concerns us is as follows:

1. Generate values of y_0 and M_Z^2 from the LO cross section for Z boson production eq. (2.1). From these variables, calculate x_a and x_b by eq. (2.2).

2. Generate a virtuality Q_1^2 for the incoming quark a, a longitudinal splitting variable fraction z_1 for the first branching, and an azimuthal angle ϕ for this branching. The distributions arise from the Sudakov form factor

$$S_i(x_a, Q^2_{\rm max}, Q_1^2) = \exp\biggl\{ - \int_{Q_1^2}^{Q^2_{\rm max}} \frac{dQ'^2}{Q'^2} \frac{\alpha_s(Q'^2)}{2\pi} \times \\ \times \sum_k \int_{x_a}^1 \frac{dz_1}{z_1}\, P_{k\to ij}(z_1) \frac{f_k(x_a/z_1, Q'^2)}{f_i(x_a, Q'^2)} \biggr\}\,. \tag{2.8}$$

Here, $Q^2_{\max}$ is normally set equal to M_Z^2. The Sudakov form factor is the probability that the virtuality of quark a is less than Q_1^2.

3. Iterate the branching for all initial-state and final-state[2] partons until no further branchings are possible.

4. Compute 4-vectors for the momenta of the generated partons.

2.3 First initial-state branching with BSZ algorithm

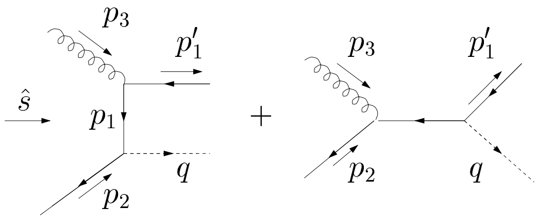

Figure 3: NLO gluon-quark collision.

Later, in section 3, we will calculate the hard-scattering cross section for the gq-induced process, figure 3. The gluon, of momentum p_3, comes from hadron A and the (anti)-quark, of momentum p_2, comes from hadron B. The NLO contribution we want to calculate is to be accurate when the incoming gluon and quark have virtualities and transverse momenta that are small compared with M_Z, and the intermediate quark, of momentum p_1, has a virtuality of order M_Z^2. To avoid double counting, it is necessary to subtract the corresponding contribution obtained from the showering algorithm applied to the LO partonic cross section, and it is this subtraction term that we calculate in this section.

The subtraction is obtained by multiplying the lowest order cross section, from eq. (2.1), by the appropriate part of the showering approximation at order α_s. Only the gluon-to-quark splitting is relevant for our calculation, and to match with the definition of the NLO hard-scattering, it should be calculated when the initial-state partons are on-shell and have zero transverse momentum, and the final-state quark is on-shell. The showering factor is just the first order term in the expansion of the Sudakov form factor [2] in powers of $\alpha_s(M_Z^2)$.

The first-order cross section in the showering approximation is

$$\frac{d\sigma_{\text{shower }1}}{dy_0\, dQ_1^2\, d\xi_3\, d\phi} = K \sum_i (A_i^2 + V_i^2) \frac{\alpha_s(M_Z^2)}{4\pi^2 Q_1^2}\, C(Q_1^2)\, P(z_1)\, \frac{1}{\xi_3} f_g(\xi_3, M_Z^2) f_{\bar{\imath}}(x_b, M_Z^2)\,. \tag{2.9}$$

Here, ξ_3 is the longitudinal momentum fraction of p_3, and the splitting kernel is for $g \to$ quark + antiquark: $P(z_1) = P_{g\to i\bar{\imath}}(z_1) = \frac{1}{2}(1 - 2z_1 + 2z_1^2)$. Because of the way in which an event generator uses the lowest-order cross section, it is the variable y_0 that appears in eq. (2.9) rather than the true rapidity of the Z boson. We will relate y_0 to the true rapidity later. Note that because we are doing a strict expansion in powers of $\alpha_s(M_Z^2)$, the scale argument of the pdf's is M_Z^2. The function $C(Q_1^2)$ is a

[2] The final-state showering is organized similarly to the initial-state showering. However, we will not need it explicitly in this paper.

cut-off function [1] that gives the maximum value of Q_1^2, and the standard choice is $C(Q_1^2) = \theta(M_Z^2 - Q_1^2)$. We will not discuss other choices of cut-off functions in this paper.

Now we reconstruct the 4-vectors for the momenta q, p_1, p_3 and p_1' of the vector boson, intermediate quark, the incoming gluon and the outgoing quark. In the Bengtsson-Sjöstrand-van Zijl's definition [2], they obey the following requirements:

1. Hadron A is to be moving in the z direction.
2. Hadron B is to be moving in the $-z$ direction.
3. The incoming partons, p_2 and p_3 have momentum fractions ξ_2 and ξ_3 relative to their parent hadrons, in the sense of light-front components.
4. $p_1^2 = -Q_1^2$.
5. $p_2^2 = p_3^2 = p_1'^2 = 0$, and p_2 and p_3 have zero transverse momentum.
6. $q^2 = M_Z^2$, $\xi_2 = x_b$.
7. $z_1 = \dfrac{(p_1+p_2)^2}{(p_3+p_2)^2} = \dfrac{x_a x_b s}{\xi_2 \xi_3 s} = \dfrac{x_a}{\xi_3}$.

In the center-of-mass frame of hadrons A and B, with the components written in the order $(p^0, \mathbf{p}_\mathrm{T}, p^z)$, we then have

$$p_1^\mu = \left(\frac{1}{2\sqrt{s}} \left[\frac{M_Z^2 + Q_1^2}{\xi_2} - \frac{Q_1^2}{\xi_3} \right], \mathbf{p}_T, \frac{1}{2\sqrt{s}} \left[\frac{M_Z^2 + Q_1^2}{\xi_2} + \frac{Q_1^2}{\xi_3} \right] \right), \tag{2.10}$$

$$p_2^\mu = \frac{\xi_2 \sqrt{s}}{2} (1, \mathbf{0}_T, -1), \tag{2.11}$$

$$p_3^\mu = \frac{\xi_3 \sqrt{s}}{2} (1, \mathbf{0}_T, 1), \tag{2.12}$$

$$p_1'^\mu = \left(p_3^0 - p_1^0, -\mathbf{p}_T, p_3^z - p_1^z \right), \tag{2.13}$$

$$q^\mu = \left(p_2^0 + p_1^0, \mathbf{p}_T, p_2^z + p_1^z \right), \tag{2.14}$$

with

$$\mathbf{p}_T = \sqrt{ -\frac{Q_1^4}{\xi_2 \xi_3 s} + Q_1^2 \left(1 - \frac{M_Z^2}{\xi_2 \xi_3 s} \right) } \, \mathbf{n}_T, \tag{2.15}$$

where $\mathbf{n}_T$ is a unit transverse vector in the direction defined by the azimuthal angle ϕ.

Note that after showering, the rapidity of the Z boson is not $y_0 \equiv \frac{1}{2} \ln(x_a/x_b)$ but is given by

$$y \equiv \frac{1}{2} \ln \frac{q^0 + q^z}{q^0 - q^z} = \frac{1}{2} \ln \frac{(Q_1^2 + M_Z^2)\, \xi_3}{\xi_2 \,(\xi_2 \xi_3 s - Q_1^2)}, \tag{2.16}$$

which does approach y_0 in the limit $Q_1 \to 0$.

The natural variables for the LO differential cross section plus the first-order showering are Q_1^2, y_0, and ξ_3. However, they are not so convenient for the NLO corrections. So we now transform the cross section in eq. (2.9) in terms of more convenient variables for a hard gluon-quark scattering: y, ξ_2, ξ_3.

From the above equations, we have

$$Q_1^2 = \frac{e^{2y} s\, \xi_2^2 - M_Z^2}{1 + e^{2y}\, \xi_2/\xi_3}\,, \tag{2.17}$$

$$y_0 = \ln \frac{M_Z}{\sqrt{s}} - \ln \xi_2\,, \tag{2.18}$$

which gives the the jacobian

$$\frac{\partial(y_0, Q_1^2)}{\partial(y, \xi_2)} = \frac{2e^{2y}\xi_3[\xi_2\xi_3 s + M_Z^2]}{(\xi_3 + e^{2y}\xi_2)^2}\,. \tag{2.19}$$

Then the cross section is

$$\frac{d\sigma^{(\mathrm{BSZ})}_{\text{shower 1}}}{dy\, d\xi_2\, d\xi_3\, d\phi} = K \sum_i (A_i^2 + V_i^2) \frac{\alpha_s(M_Z^2)}{4\pi^2 Q_1^2}\, C(Q_1^2)\, P(z_1) \times$$
$$\times\, f_g(\xi_3, M_Z^2) f_{\bar{\imath}}(\xi_2, M_Z^2) \frac{2e^{2y}[\xi_2\xi_3 s + M_Z^2]}{(\xi_3 + e^{2y}\xi_2)^2}\,. \tag{2.20}$$

2.4 First initial-state branching with New algorithm

In the standard algorithms used for treating parton kinematics, the effect of showering changes the relationship between observable quantities and the parton momentum fractions from their parton model values. In ref. [1] an algorithm was proposed that does not suffer from this effect. Here we extend this algorithm to the Drell-Yan process by requiring the rapidity y of the Z boson to be the same after we shower the incoming partons. (Thus it is not necessary to use a separate variable y_0 to denote $\frac{1}{2}\ln(x_a/x_b)$.)

The parton momenta obeys the same requirement as the BSZ algorithm, except that items 6 and 7 are replaced by

6′ $q^2 = M_Z^2$ and $y = \dfrac{1}{2} \ln \dfrac{x_a}{x_b}$.

7′ $z_1 = \dfrac{x_a}{\xi_3} \neq \dfrac{(p_1 + p_2)^2}{(p_3 + p_2)^2}$.

Thus the condition on the momentum of p_2 is dropped, and instead the rapidity of the Z boson is required to obey the simple parton-model relation to x_a and x_b. In addition, the first splitting variable z_1 is defined by 7′, without using the "$\hat{s}$"

condition. Accordingly, the fraction momentum of p_3 is required to be $\xi_3 = x_a/z_1$ in the new algorithm. Note that eqs. (2.10)–(2.16) remain true in the new algorithm. What has changed is the relation between the parton kinematics and the variable x_b that is generated by the algorithm. In the old algorithm $x_b = \xi_2$; in the new algorithm $x_b = x_a e^{-2y} = \xi_3 z_1 e^{-2y}$.

The first-order cross section in the showering approximation in the new algorithm is obtained by transforming the cross section in eq. (2.9) to be differential in the variables y, ξ_2, ξ_3, ϕ, by using eq. (2.17), which gives the relation between ξ_2 and Q_1^2:

$$\frac{d\sigma^{\rm (New)}_{\rm shower\,1}}{dy\,d\xi_2\,d\xi_3\,d\phi} = K\sum_i (A_i^2+V_i^2)\,\frac{\alpha_s(M_Z^2)}{4\pi^2 Q_1^2}\,C_1(Q_1^2)P(z_1)\times \\ \times\frac{e^{2y}\left[\xi_2\xi_3 s(2+\xi_2/\xi_3 e^{2y})+M_Z^2\right]}{(\xi_3+e^{2y}\xi_2)^2} f_g(\xi_3,M_Z^2)f_{\bar{\imath}}(x_b,M_Z^2)\,, \tag{2.21}$$

where y is the exact rapidity of Z boson. Notice that in this algorithm, although the parton density for the parton of momentum p_2 is calculated with momentum fraction $x_b \equiv e^y M_Z/\sqrt{s}$, the actual fractional momentum is different (except at zero transverse momentum), and is given by a rather complicated formula.

This difference between the actual parton momentum fraction and the value used in the evaluation of the parton density is a characteristic of this algorithm. The utility of this apparent inconsistency will become apparent when we compute the relation with the results of the "matrix element" method of calculation.

3. NLO hard cross section

3.1 Unsubtracted NLO term

From standard references (e.g., [8]), we find that the unsubtracted cross section associated with gluon-quark scattering corresponding to figure 3 is:

$$\frac{d\sigma_{\rm unsubtracted\,1}}{d\xi_2\,d\xi_3\,d\hat{t}\,d\phi} = \sum_i \frac{\alpha_s(M_Z^2)}{4\hat{s}^2}\,\frac{\sqrt{2}\,G_F M_Z^2(A_i^2+V_i^2)}{3(2\pi)}\times \\ \times\frac{\hat{s}^2+\hat{t}^2+2\hat{u}\,M_Z^2}{\hat{s}\,\hat{t}} f_g(\xi_3,M_Z^2)f_{\bar{\imath}}(\xi_2,M_Z^2)\,. \tag{3.1}$$

The sum is over quark and antiquark flavors, and

$$\begin{aligned} \hat{s} &= (p_3+p_2)^2 = \xi_2\,\xi_3 s\,, \\ \hat{t} &= -Q_1^2\,, \\ \hat{u} &= M_Z^2-\hat{t}-\hat{s} = M_Z^2+Q_1^2-\xi_2\,\xi_3 s\,. \end{aligned} \tag{3.2}$$

From eq. (2.17), we have

$$\frac{\partial \hat{t}}{\partial y} = \frac{-2e^{2y}\xi_2\,\xi_3\,[\xi_2\,\xi_3\,s + M_Z^2]}{(\xi_3 + e^{2y}\xi_2)^2}. \tag{3.3}$$

So now we can also write this unsubtracted cross section in terms of the same variables as eq. (2.21):

$$\begin{aligned}\frac{d\sigma_{\text{unsubtracted 1}}}{dy\,d\xi_2\,d\xi_3\,d\phi} = K\sum_i (A_i^2+V_i^2)\,\frac{\alpha_s(M_Z^2)}{4\pi^2 Q_1^2}\,\frac{\hat{s}^2+\hat{t}^2+2\hat{u}\,M_Z^2}{\hat{s}^2}\times\\ \times\frac{e^{2y}(\xi_2\xi_3\,s+M_Z^2)}{(\xi_3+e^{2y}\xi_2)^2}\,f_g(\xi_3,M_Z^2)f_{\bar{\imath}}(\xi_2,M_Z^2)\,,\end{aligned} \tag{3.4}$$

3.2 NLO term with subtraction

We now subtract the showering term, eq. (2.20) or (2.21), from the above $O(\alpha_s)$ term. A change in the labeling of the parton momentum fractions is in order. The variables we have previously used, ξ_2, ξ_3, etc, were tied to a particular structure for the LO hard scattering and showering. But for the subtracted NLO cross section we prefer something that has the same names for the external partons independently of the subprocess considered. So we make the following change of notation: Instead of ξ_3, we will write ξ_a to indicate that a parton from hadron A, and instead of ξ_2, we will write ξ_b to indicate that a parton from hadron B.

- For the BSZ algorithm:

$$\begin{aligned}\frac{d\sigma^{(\mathrm{BSZ})}_{\text{hard 1}}}{dy\,d\xi_a\,d\xi_b\,d\phi} = K\sum_i (A_i^2+V_i^2)\frac{\alpha_s(M_Z^2)}{-4\pi^2\hat{t}}\,f_g(\xi_a,M_Z^2)f_{\bar{\imath}}(\xi_b,M_Z^2)\times\\ \times\frac{e^{2y}[\xi_a\xi_b s+M_Z^2]}{(\xi_a+e^{2y}\xi_b)^2}\left\{\frac{\hat{s}^2+\hat{t}^2+2\hat{u}\,M_Z^2}{\hat{s}^2} - 2C_1(-\hat{t})\,P(z_1)\right\}\end{aligned} \tag{3.5}$$

- For the new algorithm:

$$\begin{aligned}\frac{d\sigma^{(\mathrm{New})}_{\text{hard 1}}}{dy\,d\xi_a\,d\xi_b\,d\phi} = K\sum_i (A_i^2+V_i^2)\frac{\alpha_s(M_Z^2)}{-4\pi^2\hat{t}}\,\frac{e^{2y}}{(\xi_a+e^{2y}\xi_b)^2}\,f_g(\xi_a,M_Z^2)\times\\ \times\left\{f_{\bar{\imath}}(\xi_b,M_Z^2)\,\frac{\hat{s}^2+\hat{t}^2+2\hat{u}\,M_Z^2}{\hat{s}^2}\,\left(\hat{s}+M_Z^2\right) - \right.\\ \left. - f_{\bar{\imath}}(x_b,M_Z^2)\,C_1(-\hat{t})\,P(z_1)\,\hat{s}\,\left(2+\frac{\xi_b}{\xi_a}\,e^{2y}+\frac{M_Z^2}{\xi_a\xi_b s}\right)\right\}.\end{aligned} \tag{3.6}$$

In both equations $\hat{s}$, $\hat{t}$, $\hat{u}$, and z_1 are all functions of ξ_a and ξ_b, given by the formulae earlier, which in terms of the new notation are:

$$\begin{aligned}-\hat{t} &= Q_1^2 = \frac{e^{2y}s\,\xi_b^2 - M_Z^2}{1+e^{2y}\,\xi_b/\xi_a}\,,\\ -\hat{u} &= \xi_a\xi_b s - M_Z^2 - Q_1^2\,,\\ z_1 &= \frac{x_a}{\xi_a}\,.\end{aligned} \tag{3.7}$$

3.3 Results for gq subprocess

Now we present the results for the other gluon-quark scattering subprocess, in which gluon comes out of hadron B instead of A, from diagrams shown in figure 4. The formulae for the subtracted cross section are obtained from eqs. (3.5) and (3.6) by making the following change:

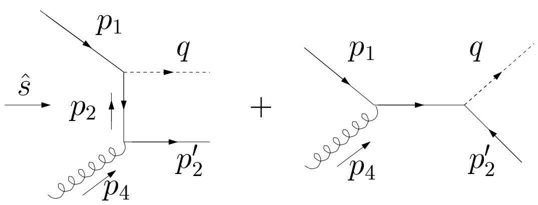

Figure 4: NLO gluon-quark collision.

$$i \leftrightarrow \bar{\imath}\,, \qquad a \leftrightarrow b\,, \qquad y \to -y\,. \tag{3.8}$$

The resulting subtracted NLO cross sections are:

- For the BSZ algorithm:

$$\frac{d\sigma^{(\mathrm{BSZ})}_{\mathrm{hard}\,2}}{dy\,d\xi_a\,d\xi_b\,d\phi} = K\sum_i (A_i^2+V_i^2)\frac{\alpha_s(M_Z^2)}{-4\pi^2\hat{u}}\, f_i(\xi_a, M_Z^2) f_g(\xi_b, M_Z^2) \times \tag{3.9}$$
$$\times\, \frac{e^{-2y}[\xi_a\xi_b s + M_Z^2]}{(\xi_b + e^{-2y}\xi_a)^2}\left\{\frac{\hat{s}^2 + \hat{u}^2 + 2\hat{t}M_Z^2}{\hat{s}^2} - 2C_2(-\hat{u})P(z_2)\right\}.$$

- For the new algorithm:

$$\frac{d\sigma^{(\mathrm{New})}_{\mathrm{hard}\,2}}{dy\,d\xi_a\,d\xi_b\,d\phi} = K\sum_i (A_i^2+V_i^2)\frac{\alpha_s(M_Z^2)}{-4\pi^2\hat{u}}\,\frac{e^{-2y}}{(\xi_b + e^{-2y}\xi_a)^2}\, f_g(\xi_b, M_Z^2) \times$$
$$\times \left\{ f_i(\xi_a, M_Z^2)\frac{\hat{s}^2 + \hat{u}^2 + 2\hat{t}\,M_Z^2}{\hat{s}^2}\left(\hat{s} + M_Z^2\right) - \right. \tag{3.10}$$
$$\left. - \, f_i(x_a, M_Z^2) C_2(-\hat{u})P(z_2)\hat{s}\left(2 + \frac{\xi_a}{\xi_b}e^{-2y} + \frac{M_Z^2}{\xi_a\xi_b s}\right)\right\}.$$

Here, $z_2 = x_b/\xi_b$.

3.4 Comparison with $\overline{\mathrm{MS}}$ scheme

As discussed in [1], after we have obtained the NLO corrections, it is necessary to find the relation between the scheme of the pdf's used in the event generator and the commonly used $\overline{\mathrm{MS}}$ scheme.[3] As in [1] we will compare the same cross

[3]The need for the change of scheme is not apparent in other work on merging parton showers and matrix elements [6]. Our work relies on a deeper analysis of the derivation of the whole algorithm used in a Monte-Carlo event generator, rather than just a consideration of the normalization of a particular inclusive cross section. It is only in this context that it becomes apparent that the $\overline{\mathrm{MS}}$ parton densities are not the appropriate ones. The parton densities used in an event generator implicitly include observed jets that result from initial-state showering, and therefore the definition of the parton density is directly tied to the definition of the showering algorithm. One way to ensure that the pdf's for the event generator are the same as $\overline{\mathrm{MS}}$ pdf's is to adjust the cut off function C suitably, as is done by Pötter [9].

section computed with the $\overline{\mathrm{MS}}$ pdf's and with the formulae used in the Monte-Carlo approach. We will choose to calculate $d\sigma/dy$, since

- This cross section can be calculated analytically both in the normal factorization method and from the algorithm used in the event generator. (The inclusive cross section allows an integral over the showering probabilities.)
- The lowest order has a factorized dependence on both parton momentum fractions, so there is exactly enough information to extract the parton densities.

As already discussed in [1], the unsubtracted part of the cross section is the same in both methods of calculation, so that the result of the calculation depends only on the subtraction terms. The relation between the pdf's is therefore independent of the details of the electro-weak couplings for example.

It is less obvious that the relation between the pdf's could be unaffected by the differences between the kinematics for the DIS and DY processes. This is what we will find, nevertheless.

In order to get a complete determination of the parton densities, we consider a modified cross section, in which only a single quark and antiquark flavor annihilate, and in which M_Z is given an arbitrary value. Moreover, the electroweak couplings form a common factor, and it is convenient to remove them and to arrange that the lowest order cross section is just $f_i(x_a)f_{\bar{\imath}}(x_b)$; this we call the structure function $\mathcal{F}_i(y, M_Z^2)$. First we take the formula for the structure function in the $\overline{\mathrm{MS}}$ scheme [10]:

$$
\begin{aligned}
\mathcal{F}_i(y, M_Z^2) &= f_i^{(\overline{\mathrm{MS}})}(x_a,\mu^2) f_{\bar{\imath}}^{(\overline{\mathrm{MS}})}(x_b,\mu^2) + \\
&+ \frac{\alpha_s(\mu^2)}{2\pi} \int_{x_a}^1 d\xi_a \int_{x_b}^1 d\xi_b \Bigg\{ f_g^{(\overline{\mathrm{MS}})}(\xi_a,\mu^2) \times \\
&\times \Bigg[f_{\bar{\imath}}^{(\overline{\mathrm{MS}})}(x_b,\mu^2)\, \delta(\xi_b - x_b)\, \frac{1}{\xi_a} \left(P(z_1) \ln \frac{2(\xi_a - x_a)(1-x_b)}{x_b(\xi_a + x_a)} + z_1(1-z_1) \right) + \\
&\quad + f_{\bar{\imath}}^{(\overline{\mathrm{MS}})}(\xi_b,\mu^2) \left(\frac{G^c(\xi_a,\xi_b)}{\xi_b - x_b} + H^c(\xi_a,\xi_b) \right) - f_{\bar{\imath}}^{(\overline{\mathrm{MS}})}(x_b,\mu^2) \frac{G^c(\xi_a,x_b)}{\xi_b - x_b} \Bigg] + \\
&\quad + (\bar{\imath} \leftrightarrow i,\ a \leftrightarrow b,\ y \leftrightarrow -y) \Bigg\} + \text{first-order quark terms} + O(\alpha_s^2)
\end{aligned}
\tag{3.11}
$$

with

$$
G^c(\xi_a,\xi_b) = \frac{x_b(\tau + \xi_a\,\xi_b)\,[\tau^2 + (\tau - \xi_a\,\xi_b)^2]}{\xi_a^3\,\xi_b^2\,(\xi_a\,x_b + \xi_b\,x_a)(\xi_b + x_b)}\,, \tag{3.12}
$$

and

$$
H^c(\xi_a,\xi_b) = \frac{\tau(\tau + \xi_a\,\xi_b)\,[\xi_a\,\xi_b^2\,x_a + \tau(\xi_a x_b + 2\,\xi_b\,x_a)]}{(\xi_a\,\xi_b)^2\,(\xi_a\,x_b + \xi_b\,x_a)^3}\,. \tag{3.13}
$$

Here the variables x_a and x_b are defined to be exactly the "parton-model" values $x_a = \sqrt{\tau}e^y$ and $x_b = \sqrt{\tau}e^{-y}$, with $\tau = M_Z^2/s$. The variable z_1 in the splitting kernel is defined as $z_1 = x_a/\xi_a$, the same as in the new algorithm.

This structure function must equal the same structure function given by the Monte-Carlo calculation. Since the expressions from the new algorithm are simpler and have similar structure, we will first compare with the structure obtained using the new algorithm.

We start from eq. (3.6) for the cross section. Using the functions defined in eq. (3.12) and eq. (3.13), we rewrite so as to obtain a formula with a similar structure to the $\overline{\mathrm{MS}}$ formula:

$$\begin{aligned}\mathcal{F}_i(y, M_Z^2) = {} & f_i^{(\mathrm{new})}(x_a, M_Z^2) f_{\bar\imath}^{(\mathrm{new})}(x_b, M_Z^2) + \\ & + \frac{\alpha_s(M_Z^2)}{2\pi} \int_{x_a}^1 d\xi_a \int_{x_b}^1 d\xi_b \Bigg\{ f_g^{(\mathrm{new})}(\xi_a, M_Z^2) f_{\bar\imath}^{(\mathrm{new})}(x_b, M_Z^2) \times \\ & \times \delta(\xi_b - x_b) C_1(Q_1^2)\, P(z_1) \frac{\xi_b^2\,(\xi_a^2 - x_a^2) - (\tau + \xi_a\,\xi_b)^2}{x_a \xi_a\,(\xi_b^2 - x_b^2)\,(\xi_a x_b + \xi_b x_a)} + \\ & + f_g^{(\mathrm{new})}(\xi_a, M_Z^2) f_{\bar\imath}^{(\mathrm{new})}(\xi_b, M_Z^2) \left[\frac{G^c(\xi_a, \xi_b)}{\xi_b - x_b} + H^c(\xi_a, \xi_b) \right] + \\ & + (\bar\imath \leftrightarrow i,\, a \leftrightarrow b,\, y \leftrightarrow -y) \Bigg\} + \\ & + \text{first-order quark terms} + O(\alpha_s^2)\,. \end{aligned} \tag{3.14}$$

Comparison of eq. (3.14) and eq. (3.11) shows that

$$\begin{aligned} x f_a^{(\mathrm{new})}(x, M_Z^2) = {} & x f_a^{(\overline{\mathrm{MS}})}(x, M_Z^2) + \\ & + \frac{\alpha_s(M_Z^2)}{2\pi} \int_x^1 d\xi \frac{x}{\xi} f_g^{(\overline{\mathrm{MS}})}(\xi, M_Z^2)\, [P(z) \ln(1-z) + z(1-z)] + \\ & + \text{first-order quark terms} + O(\alpha_s^2)\,, \end{aligned} \tag{3.15}$$

and $z = x/\xi$, the same as in ref. [1].

When we attempt a similar calculation with the BSZ algorithm, we find that the parton densities in the subtraction term in the equivalent of eq. (3.14) are no longer a product of $f_g(\xi_a)$ and $f_{\bar\imath}(x_b)$. Instead the argument, $x_b = \sqrt{\tau} e^{-y}$, of the second parton density is replaced by a complicated function of the kinematic variables. This does not match the structure of the $\overline{\mathrm{MS}}$ formula eq. (3.11), and it is not possible in any simple way to extract a relation between the parton densities for the BSZ algorithm and for the $\overline{\mathrm{MS}}$ scheme. The problem is that the showering of the parton on the A side has affected the kinematics of the parton on the B side and that it is the modified kinematics that are used in the corresponding parton density. As far as we can see, a correct analysis can only be done by investigating the problem in terms of unintegrated parton densities. Since this is a much more complicated problem, we shall not attempt it here.

4. W production

In the above sections we described only Z production since the formulae are for annihilation of a quark i with its antiquark $\bar\imath$. For W production the results are similar, only that one needs a different flavor of antiquark $\bar{\imath}'$ and an appropriate change in the overall coupling:

$$\sqrt{2}\, G_F\, (A_i^2 + V_i^2)\, M_Z^2 \longrightarrow \sqrt{2}\, G_F\, |V_{ii'}|^2\, M_W^2\,, \tag{4.1}$$

where $V_{ii'}$ is an element of CKM matrix.

5. Conclusion

We showed how to incorporate the gluon-quark processes in a Monte-Carlo event generator using the subtraction method proposed by Collins in ref. [1]. We also analyzed the exact parton kinematics used in the BSZ algorithm, and observed that the factorization theorem for the Drell-Yan process is used in a modified form compared with the form normally used for the inclusive cross section $d\sigma/dy$.

When we computed the relation between the parton densities for the event generator and the $\overline{\text{MS}}$ densities, we found the same relation as found in [1] in the context of deep-inelastic scattering, but only if we used the new algorithm for parton kinematics that was proposed in [1]. This supports the hypothesis of process independence of the pdf's. For the case of the normal BSZ algorithm, we found that the effect of the correlated parton kinematics appears not to permit us to obtain a simple relation.

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